MATH 111  RATIONAL FUNCTIONS  REVIEW

1. Specify the domain of the rational function \( f(x) = \frac{x^2 - 5x}{x^3 + 3x + 2} = \frac{x(x-5)}{(x+1)(x+2)} \)

   domain: \( x \neq -1, -2 \)

\[ (\infty, -2) \cup (-2, -1) \cup (-1, \infty) \]

2. Give the \textbf{equation(s)} of the vertical asymptotes for the function \( f(x) = \frac{x^2 - 3x - 4}{2x^2 - 4} = \frac{(x-4)(x+1)}{2(x+2)(x-2)} \)

   and show the analysis that leads to your conclusion.

   \[ \text{vertical asymptote} \]

   \[ x = \sqrt{2} \quad \text{and} \quad x = -\sqrt{2} \]

3. Find the \textbf{equation} of the horizontal asymptote(s) for the function \( f(x) = \frac{2x^3 - 4x}{5x^3 + 7x^2} \) and show the analysis that leads to your conclusion.

   \[ \text{horizontal asymptote} \quad y = \frac{2}{5} \]

4. Find the \textbf{equation} of the slant asymptote for the function \( f(x) = \frac{x^3 - x^2 - 4x + 5}{x^2 + x + 6} \). Show your work.

   \[ x^2 + x + 6 \left[ \frac{x - 2}{x^3 - x^2 - 4x + 5} \right] \]

   \[ \frac{x^3 + x^2 + 6x}{-2x^2 - 10x + 5 - 2x^2 - 2x + 2} \]

   \[ \text{slant asymptote} \quad y = x - 2 \]
5. Determine a rational function that satisfies the following conditions and sketch the graph:

i) Has vertical asymptotes at \( x = 2 \) and \( x = -1 \)
ii) Has a horizontal asymptote \( y = 2 \)
iii) Has x-intercepts at -3 and 4

\[
f(x) = \frac{2(x+3)(x-4)}{(x-2)(x+1)}
\]

6. Sketch the graph of a rational function with the following conditions:

i) has \( f(4) = 0 \)
ii) \( f(x) \to 2 \) as \( x \to \infty \)
iii) \( f(x) \to -\infty \) as \( x \to 3^+ \) and \( f(x) \to \infty \) as \( x \to 3^- \)
iv) has y-axis symmetry

\[
f(x) = \frac{2(x+4)(x-4)}{(x-3)(x+3)}
\]